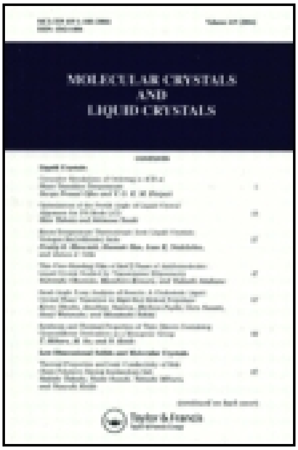

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M. Sugi [a] , Y. Hirano [a] , Y. F. Miura [a] & K. Saito [b]

[a] Toin University of Yokohama, Aoba-ku, Yokohama, 225-8502, JAPAN

[b] Electrotechnical Laboratory, Tsukuba-shi, Ibaraki, 305-8568, JAPAN

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Optimization of Fractal Immittance Networks

M. SUGI[a], Y. HIRANO[a], Y.F. MIURA[a] and K. SAITO[b]

[a]*Toin University of Yokohama, Aoba-ku, Yokohama 225–8502, JAPAN and*
[b]*Electrotechnical Laboratory, Tsukuba-shi, Ibaraki 305–8568, JAPAN*

Fractal immittance, $Y(\omega)\propto\omega^{a}$ or $Z(\omega)\propto\omega^{-a}$ ($0<a<1$), is simulated by the analogue circuits composed of resistors R and capacitors C based on the distributed-relaxation-time (DRT) models. The optimum interval of relaxation times to give the widest bandwidth is searched by the numerical calculation. It is found that the bandwidth of 5 decades with a phase-angle error, e.g., of ±0.5° for a =0.25, 0.5 and 0.75 can be realized using twenty or less R-C pairs.

Keywords: fractal; fractal immittance; fractance; computer simulation

INTRODUCTION

The power-law conductivity (PLC), $\sigma \propto \omega^{a}\{\cos(a\pi/2)+j\sin(a\pi/2)\}$ ($0<a<1$) with a constant phase angle $a\pi/2$, has been observed in various material systems[1]. The PLC behavior, often ascribed to the fractal distribution of the time constants of dielectric relaxation process, is found to represent a non-integer-rank differential/integral (NIDI) operator. It leads to a unique memory effect: the one-to-one correspondence is held between the input in the past and the output in the future, forming a new category of circuit elements other than the conventional elements, the resistor R, the capacitor C and the inductor L[2]-[7]. This category is now known as the power-law-case diode[2],[3],[5], the fractance[4] or the fractal immittance diode (FD)[7],[8].

There will be two possible ways to the hardware realization of NIDI operator or FD. One is to fabricate FD using PLC of appropriate material systems. The values of a are however confined to $a \approx 0.8$ in most cases, and it is indispensable to develop the means to modify the distribution of the time constants in order to obtain an optional a-value.

The other is to simulate NIDI action by analogue networks consisting of a large number of conventional elements[6],[10],[11]. In previous papers, we have shown that FD or NIDI operator with an optional a-value, positive and negative as well ($0<|a|<1$), can be simulated by analogue networks consisting of R-C or L-C pairs[6].

In both ways, it is therefore useful to find out the minimum number of time constants in the materials or that of the element pairs in the optimized networks for FD against a given criterion. We have examined the optimum conditions by the numerical calculation employing a computer. The results are shown in the present paper.

OPTIMIZATION OF FD EQUIVALENT CIRCUITS

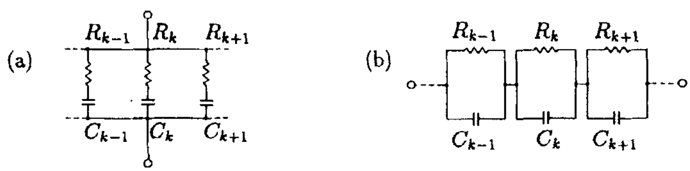

FIGURE 1 Equivalent circuits of FD admittance (a) and FD impedance (b) according to DRT models.

According to the distributed-relaxation-time (DRT) model, the admittance and the impedance of FD are given by

$$Y(\omega) = \frac{Y_F(a)\sin a\pi}{\pi}\int_{-\infty}^{+\infty}\frac{j\omega\lambda^{a}}{\lambda + j\omega}d(\ln\lambda) \tag{1}$$

and

$$Z(\omega) = \frac{Z_F(a)\sin a\pi}{\pi}\int_{-\infty}^{+\infty}\frac{\lambda^{1-a}}{\lambda + j\omega}d(\ln\lambda) \tag{2}$$

respectively, each represented by Figs.1(a) and 1(b), where λ^{-1} refers to the time constant. If we approximate Eqs.(1) and (2) by finite terms, we obtain

$$Y_n(\omega) = \frac{Y_{Fn}(a)\sin a\pi}{\pi}\sum_{k=1}^{n}\frac{j\omega\lambda_k^{\,a}}{\lambda_k + j\omega}\Delta(\ln\lambda) \tag{3}$$

and

$$Z_n(\omega) = \frac{Z_{Fn}(a)\sin a\pi}{\pi}\sum_{k=1}^{n}\frac{\lambda_k^{\,1-a}}{\lambda_k + j\omega}\Delta(\ln\lambda) \tag{4}$$

where $\Delta(\ln\lambda)=\ln\lambda_{k+1} - \ln\lambda_k$ is the pole interval taken in the logarithmic scale, and $Y_{Fn}(a)$ and $Z_{Fn}(a)$ are the coefficients for adjusting the magnitudes.

The constant phase angle $|\theta|=a\pi/2$ is approximated as

$$\theta = \arctan\left\{\frac{\operatorname{Im} Y_n(\omega)}{\operatorname{Re} Y_n(\omega)}\right\} \tag{5}$$

for FD admittance, and

$$\theta = \arctan\left\{\frac{\operatorname{Im} Z_n(\omega)}{\operatorname{Re} Z_n(\omega)}\right\} \tag{6}$$

for FD impedance, respectively.

The optimum pole interval $\Delta(\log\lambda)_{op}$ and the optimum bandwidth B_{op} in the logarithmic scale can be numerically evaluated for each equivalent circuit with n pairs of R_k and C_k (with $R_kC_k=\lambda_k^{-1}$) as such that gives the largest single-domain band which fulfils a criterion with respect to the phase angle $|\Delta\theta|=|(|\theta|-a\pi/2)|\leq\varepsilon$ for a given value of deviation ε. It is easily demonstrated that $Y_n(\omega)$ and $Z_n(\omega)$ with the values of exponent a and $1-a$ share one and the same set of $\Delta(\log\lambda)_{op}$ and B_{op}. This relation is useful for saving the calculation time.

It has been found that $\Delta(\log\lambda)_{op}$ for given a and ε fluctuates as n increases up to a critical value $n=n_c$ and then remains constant independent of n, while B_{op} for $n>n_c$ is found to be approximated as $B_{op}=B_0+B_1n$ or $B_{op}=B_1(n-n_0)$ with B_1 identical with the constant $\Delta(\log\lambda)_{op}$ and n_0 appreciably smaller than n_c.

The minimum number of R-C pairs n_{min} required for a given value of B_{op} is estimated from the above mentioned calculation. Table I exemplifies n_{min} and $\Delta(\log\lambda)_{op}$ for $B_{op}\geq5$ for a=0.25, 0.5 and 0.75 with ε=2°, 1°, 1/2°, 1/4° and 1/8° together with B_{op} to be realized. For each a-value, n_{min} and $\Delta(\log\lambda)_{op}$ are decreasing and increasing functions of ε., respectively, and the condition of

$B_{op} \geq 5$ is satisfied with thirty or less R-C pairs.

It is mentioned that thus obtained optimum values, n_{min}, $\Delta(\log\lambda)_{op}$ and B_{op}, can be also referred to as the numbers of the time constants, their interval of occurrence and the bandwidth to be realized, respectively, of the material systems, and that these values are directly appplicable to the cases of negative a if we take $|a|$ instead. The details will be shown on later[11].

TABLE I Minimum numbers of R-C pairs n_{min} required for $B_{op} \geq 5$ and the corresponding $\Delta(\log\lambda)_{op}$.

a=0.25 or 0.75				a=0.5			
ε	n_{min}	$\Delta(\log\lambda)_{op}$	B_{op}	ε	n_{min}	$\Delta(\log\lambda)_{op}$	B_{op}
2°	12	1.091	5.718	2°	11	0.954	5.804
1°	16	0.914	5.756	1°	15	0.879	5.532
1/2°	20	0.756	5.540	1/2°	18	0.754	5.458
1/4°	24	0.650	5.216	1/4°	23	0.680	5.646
1/8°	30	0.627	5.218	1/8°	26	0.584	5.354

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